SIMPLE HARMONIC MOTION

OBJECTIVE:

Investigating the concept of the spring constant *k* as a measure of the stiffness of a spring.
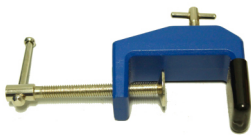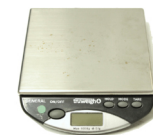
MATERIALS:

Table Clamp

Stopwatch

Digital Scale

Hooked Weights

Pendulum Clamp

Meter Stick

Rod

Spring

Ruler

CONCEPT:

THINK: What do you imagine when you read "Harmonic Motion"?

In a nutshell, it describes any repeating, back and forth oscillation over a consistent period of time. It is easily recognizable because it creates a sinusoidal curve when plotting distance vs time. Engineers love things that produce this curve because it is easy to predict their behavior and then develop technology around it. Indeed, this concept has been applied in numerous technologies and can be seen in natural life, too. See the *Real World Applications* section below for examples! To observe this in our lab we will use springs, which provide a useful model of this oscillatory behavior.

In 1660 (360+ years ago!), Robert Hooke developed his famous Hooke's Law, which describes the "restoring force" observed when a stretched or compressed spring is released, see Figure (1). Hooke found that this force is
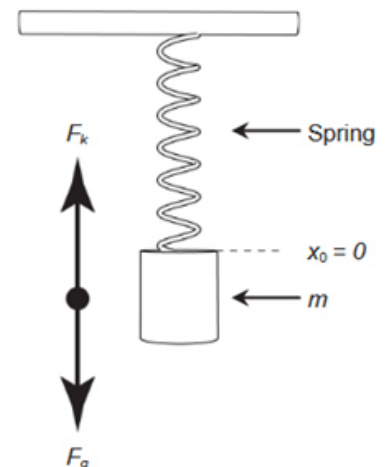

Figure 1

<u>proportional</u> to the total distance traveled, *Δy*, from equilibrium to the max stretched or compressed position. Thus, *a 25 year old* gave us the first step to understanding simple harmonic motion. Not bad!

If a mass *m* is attached to one end of a hanging spring, Hooke's spring force is equal to the gravitational force (Newton's second law) at the equilibrium position:

$$F = -k\Delta y = ma \tag{1}$$

where *k* is the stiffness constant. Realize that **the motion around a circle can be plotted across time to create a sine wave, see Figure (2)**. This means we can use <u>angular frequency</u> *ω* to calculate the frequency or velocity of an oscillation:

$$\omega = \frac{\Delta\theta}{\Delta t} = 2\pi f = \frac{2\pi}{T} \tag{2}$$

Since *ω* is part of the <u>harmonic motion equations</u>, we can solve Equation (1) for acceleration *a* and derive:

$$\omega = \sqrt{\frac{k}{m}} \tag{3} \qquad T = 2\pi\sqrt{\frac{m}{k}} \tag{4}$$

Now, let's also consider a fraction of the mass of the spring fm_s in the oscillation to improve this equation as:
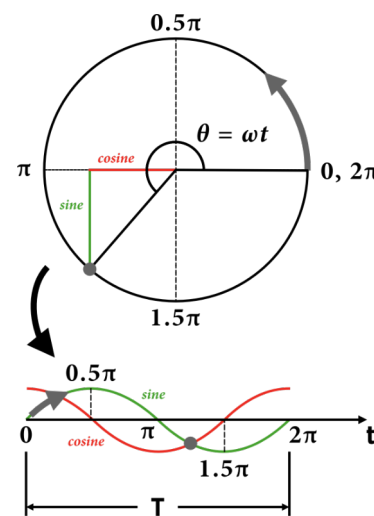
$$T = 2\pi\sqrt{\frac{m + fm_s}{k}} \tag{5}$$

$$T^2 = \left(\frac{4\pi^2}{k}\right)m + \left(\frac{4\pi^2 fm_s}{k}\right) \tag{6}$$

Notice anything familiar about Equation (6)? Compare that to a linear equation ($y = mx + b$). The plot of T^2 vs *m* has a linear relationship with stiffness! Therefore, **the period of the system is related to how stiff the oscillator is**:

$$\text{x, y: } m, T^2 \qquad \text{Slope} = \left(\frac{4\pi^2}{k}\right) \text{ and y-intercept} = \left(\frac{4\pi^2 fm_s}{k}\right)$$

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k!*

Figure 2

π is a universal constant; no matter the size curve, it can be a universal position "marker" along any periodic curve. Since the sine curve is periodic, we can use *π* to derive the change in position and the period *T* for angular velocity/frequency *ω*.

Real World Applications

- Harmonic oscillations are *everywhere!* They are the source of virtually all sinusoidal vibrations and waves, but it is very difficult to find a perfect simple harmonic oscillator in nature; most oscillators are **damped**.

- Some examples of harmonic oscillators are a bouncing ball, the shock absorbers in your car, electronic circuit clocks, stringed musical instruments, the pendulum motion of a play swing, the tides of the oceans and lakes, and even planetary motion.

- The harmonic oscillator potential is used by engineers to create models of many physical phenomena.

1) Combining Fourier math and harmonic motions to create awesome!

2) Quantum springs!

PRECAUTIONS:

A falling mass can cause injury. *Be cautious of things under the mass and only pull on the spring–mass in the manner described below!*

PROCEDURE:

Part 1

1. ☐ Fill out the top information on the worksheet.

2. ☐ REQUIRED: Read the *Concept* section.

3. ☐ Assemble the setup as shown in Figure (3).

   3.1. Make sure the spring is screwed tight under the pendulum clamp, see Figure (3b).

   3.2. Pick a mass between 200g and 800g (overall you need to try it for six different masses) and hang it on the spring. You can couple the mass as shown in Figure (3c).
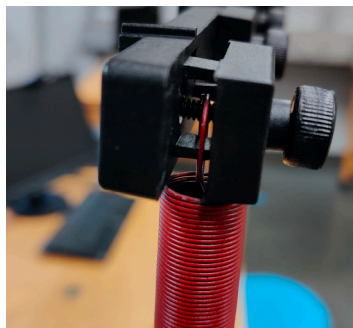
| (a) | (b) | (c) |

Figure 3

4. ☐ Record the color and mass of your spring on your worksheet under "Part 1."

5. ☐ Pull the mass down 3cm *very precisely* using the meter stick and let the mass start oscillating.

6. ☐ While the mass is oscillating and at the *exact* moment the mass reaches the bottom of its travel, start timing the oscillation for 20 full periods.

7. ☐ Now that you have the overall time for the 20 oscillations, divide the overall time by the number of the oscillations to find the Period T (s) for each oscillation.

8. ☐ Repeat steps 2-5 for six different masses and fill out Table 1 on the worksheet.

9. ☐ Answer Questions 1 & 2 on the worksheet.

IDEA TO REMEMBER!

Springs have a constant stiffness, called k!

CONCEPT & PROCEDURE VIDEOS:

Part 2

1. ☐   Measure the initial length of the spring <u>without</u> any mass attached.

2. ☐   Hang one of your selected masses on the spring and measure the length of the spring.

3. ☐   Fill out the mass m (kg) and displacement y (m) on Table 2 on the worksheet. Remember that displacement is the difference of the stretched length minus the initial length.

4. ☐   Use Newton's second law of motion to calculate the force F (N) on the spring for Table 2.

5. ☐   Repeat step 2-4 for all six masses you selected in Part 1.

6. ☐   Plot F (N) vs Δy (m) using Excel, graph paper, or the back of your worksheet, and ask your TA to sign off on your worksheet that they saw your plot.

7. ☐   Answer Questions 3 & 4 on the worksheet and follow the **Let's THINK!** instructions below.

> **IDEA TO REMEMBER!**
>
> Springs have a constant stiffness, called *k!*

Let's THINK!

- **Ask questions**: What are you learning here?... Why is this Physics concept important and how can it be used?... What do you not understand?... (For more information on this Physics topic, scan the QR codes in the *Real World Applications* and at the start of the *Procedure* section.)

- **Discuss** the concept and demonstration with your partner to help each other understand better. Discussion makes learning active instead of passive!

- For **FULL PARTICIPATION [15 points]** you must call on the TA when you have finished your group discussion to answer some comprehensive questions. If you do not fully understand and the TA asks you to discuss more, you must call on them one more time to be dismissed with full marks.

- **CONCLUSION [10 points]**: In the Conclusion section at the end of the worksheet, write 3 or more sentences summarizing this concept, how this lab helped you understand the concept better, and the real world implications you see. Do you still have questions? If so, write those as well.

Updated Date	Personnel	Notes
2022.08	Chase Boone, Ahmad Sohani, Bernard Osei, Brooks Olree	2022 Summer Improvement: Created new format.
2022.08, 2023.01	Chase Boone	Corrections and clarifications.

Name: _____   *PH1123* Section #:_____

Name: _____   TA Name:_____

SIMPLE HARMONIC MOTION
WORKSHEET [70 points]

IDEA TO REMEMBER!
Springs have a constant stiffness, called *k!*

Part 1

Color of Spring: _____ [1 point]

Mass of Spring: _____ kg [1 point]

Table 1: Squared Period of Oscillation vs Mass [12 points]

Mass of hooked weights m (kg)	Time elapsed for 20 oscillations t (s)	Period of oscillation $T(s) = \dfrac{t}{20}$	Squared period of oscillation T^2 (s²)

1) Use Excel to plot your data from Table 1 (T^2 vs m), then find your slope and y-intercept. From there, calculate your stiffness constant k and effective mass fm_s of your spring. (Show your calculations.) [15 points]

   TA Signature/Initials: _____

2) What are the possible errors in this method? [4 points]

Part 2

Initial length: _____ (m)   [1 point]

Table 2: Force vs Displacement [10 points]

Mass of hooked weights m (kg)	Displacement from compressed to hanging equilibrium y (m)	Weight due to m F (N)

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k!*

3) Using an Excel Scatter graph and Trendline, plot F vs Δy from Table 2 to find the slope. Why did we plot F vs Δy? [10 points]

TA Signature/Initials: _____

4) How different was your result between the two methods? Which calculation for the spring constant would you believe to be more reliable, and why? [6 points]

Conclusion

Write 3 or more sentences summarizing this concept, how this lab helped you understand the concept better, and the real world implications you see. Do you still have questions? If so, write those here as well. [10 points]

IDEA TO REMEMBER!

Springs have a constant stiffness, called *k!*